INSTABILITY OF OSCILLATIONS IN CABLE-STAYED BRIDGES*

Josef Malík, Ostrava

(Received July 4, 2003, in revised version September 30, 2003)

Abstract. In this paper the stability of two basic types of cable stayed bridges, suspended by one or two rows of cables, is studied. Two linearized models of the center span describing the vertical and torsional oscillations are investigated. After the analysis of these models, a stability criterion is formulated. The criterion expresses a relation between the eigenvalues of the vertical and torsional oscillations of the center span. The continuous dependence of the eigenvalues on some data is studied and a stability problem for the center span is formulated. The existence of a solution to the stability problem is proved. Some other qualitative results concerning the stability/instability of oscillations are studied as well.

Keywords: cable stayed bridge, vertical and torsional oscillations, eigenvalues and eigenfunctions of center span

MSC 2000: 35P10, 35Q80, 35B27

1. Introduction

Cable stayed bridges are quite popular because of their economic efficiency and relatively easy way of erection. The characteristic feature of the cable stayed bridge construction is the fact that road beds are suspended by one or two rows of cables which are fixed to pylons. The construction of cable stayed bridges is similar to the construction of suspension bridges whose road bed is suspended by cables as well, but these cables are not fixed to pylons but to main cables and the main cables are fixed to the pylons. Suspension bridges have been studied in many papers, for instance [1]–[5], [7]–[11], [16], where main attention has been paid to vertical oscillation of the center span, which is the part of the road bed between the pylons. The center span is modeled as a bending beam suspended by nonlinear strings.

In [12], [13] two models of the center span suspended by one and two rows of cables were studied. The models include vertical and torsional oscillations and the

* This work was supported by Grant ASCR K1019101.

nonlinear behaviour of the cables. The two basic types of cable-stayed bridges are depicted in Figs. 1 and 2.

Vertical and torsional oscillations in the models are connected through the cables and the effect of wind. The effect of wind generally depends not only on its power but also on the torsional twisting of the center span. The non-linearity of cables is as follows: the restoring force due to a cable is such that it resists expansion, but does not resist compression. The oscillations of the center span from small to medium amplitudes are linear, which means that the cables do not loosen.

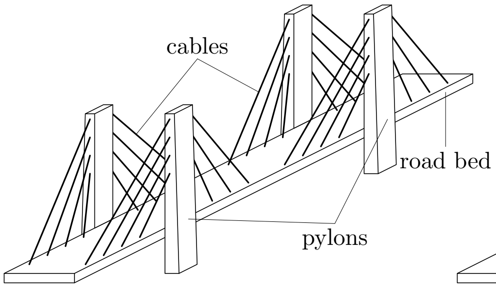

Figure 1.

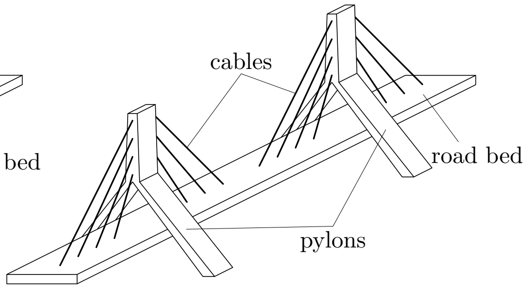

Figure 2.

In this paper we deal with some simplified models which lead to a linear system of differential equations. After analyzing the equations, we formulate a criterion of stability. The criterion is based on certain relations between the eigenvalues corresponding to the vertical and torsional oscillations of the center span. Then we study the continuous dependence of the eigenvalues on some bridge parameters (position of cables, additional loads).

A homogenization procedure, which makes it possible to describe the cable system with a usual function, is proposed. Relations between the eigenvalues of the homogenized problem and the original problem are studied. An optimal problem which makes it possible to maximize the stability of the center span in wind with respect to the parameters mentioned above is formulated. Some qualitative results concerning the relation between the length and the stability of the center span are discussed as well as some connections between the construction of the center span, the way the center span is suspended, and the stability of the whole system.

2. Setting of the basic problems

In this section we will formulate two basic models which describe the behaviour of the center span suspended by one and two rows of cables. The reader can find more information about the models in [12], [13].

The models depicted in Figs. 3 and 4 correspond to the cable-stayed bridges in Figs. 1 and 2. The models describe the behaviour of the center span in wind properly if the pylons are sufficiently stiff. The behaviour of the center span can be described by two functions $u(x,t)$, $\varphi(x,t)$ defined on $\langle 0,L\rangle \times \langle 0,T\rangle$. The function $u(x,t)$ corresponds to the downward bending of the longitudinal axis at the position x and the time t and $\varphi(x,t)$ corresponds to the torsional twisting of the cross section at x and t, which is depicted in Fig. 5.

Let us define bilinear forms

$$m_1(u,v) = \int_0^L M_1 uv \,\mathrm{d}x, \quad m_2(\varphi,\psi) = \int_0^L M_2 \varphi\psi \,\mathrm{d}x,$$
$$k_1(u,v) = \int_0^L K_1 \frac{\partial^2 u}{\partial x^2}\frac{\partial^2 v}{\partial x^2}\,\mathrm{d}x, \quad k_2(\varphi,\psi) = \int_0^L K_2 \frac{\partial \varphi}{\partial x}\frac{\partial \psi}{\partial x}\,\mathrm{d}x,$$
$$(u,v) = \int_0^L uv\,\mathrm{d}x, \quad b(u,v) = \sum_{i=1}^{8} k_i u(x_i)v(x_i),$$

where M_1, M_2, K_1, $K_2 \in L^\infty(\!(0,L)\!)$, $k_i \in \mathbb{R}$, $k_i > 0$, $i = 1,\dots 8$, and the assumptions

$$M_1(x) > \varepsilon, \quad M_2(x) > \varepsilon, \quad K_1(x) > \varepsilon, \quad K_2(x) > \varepsilon \tag{2.1}$$

are satisfied for all $x \in (0,L)$, where ε is a positive constant. The bilinear forms $k_1(\cdot,\cdot)$, $k_2(\cdot,\cdot)$ are connected with the deformation energy of the bending and twisting of the central span.

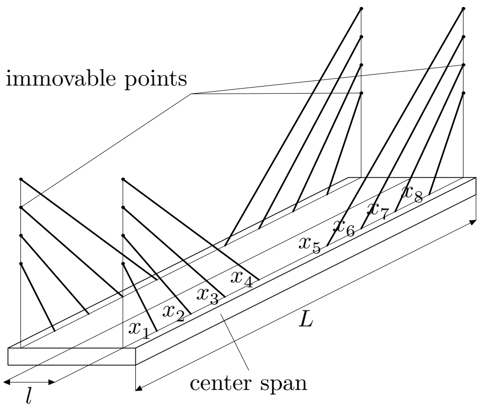

Figure 3.

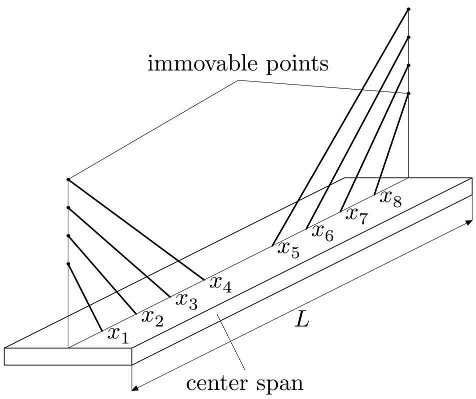

Figure 4.

The bilinear form $b(\cdot,\cdot)$ corresponds to the deformation energy of one row of cables as it is depicted in Figs. 3 and 4, where x_i are the points at which the cables are fixed to the center span. The bilinear forms $m_1(\cdot,\cdot)$, $m_2(\cdot,\cdot)$ are connected with the kinetic energy of vertical and torsional oscillations and the functions M_1, M_2 are defined as follows:

$$M_1(x) = \int_{A_x} \varrho(x,y,z)\,\mathrm{d}y\,\mathrm{d}z, \quad M_2(x) = \int_{A_x} \varrho(x,y,z)(y^2+z^2)\,\mathrm{d}y\,\mathrm{d}z, \tag{2.2}$$

where $\varrho(x,y,z)$ is the density of the center span and A_x is the cross section at x perpendicular to the longitudinal axis (see Fig. 5).

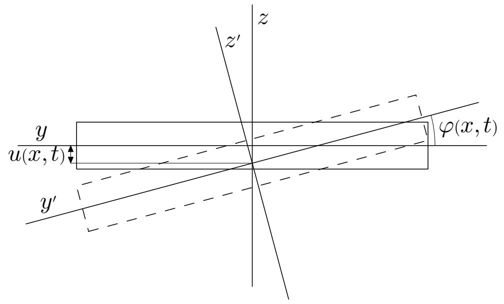

Figure 5.

We can see that $M_1(x)$ corresponds to the mass of the unit length of the center span at x and $M_2(x)$ to the cross-section moment of inertia with respect to the longitudinal axis at x.

Let us formulate variational equations for the two basic types of cable-stayed bridges. We start with the equations describing the behaviour of the center span suspended by two rows of cables. The equations read

$$\begin{aligned} m_1\Big(\frac{\partial^2 u}{\partial t^2}, v\Big) + k_1(u,v) + b((u+l\varphi)^+, v) + b((u-l\varphi)^+, v)& \\ = (F_1 + P_1(\varphi), v)&, \\ m_2\Big(\frac{\partial^2 \varphi}{\partial t^2}, \psi\Big) + k_2(\varphi,\psi) + lb((u+l\varphi)^+, \psi) - lb((u-l\varphi)^+, \psi)& \\ = (F_2 + P_2(\varphi), \psi)&, \end{aligned} \tag{2.3}$$

where u, φ are sufficiently smooth and defined on $\langle 0,L\rangle \times \langle 0,T\rangle$ and the functions v, ψ are sufficiently smooth and defined on $\langle 0,L\rangle$. The symbol u^+ stands for $\max\{0,u\}$ and corresponds to the process of loosening cables which occurs if the center span achieves a certain height at the points, where the cables are fixed. The number l is the distance between the rows of cables and the axis (see Fig. 3).

The equations (2.3) are fulfilled for all $t \in \langle 0,T\rangle$ and v, ψ and the following boundary and initial conditions

$$
\begin{aligned}
u(0,t) = u(L,t) = \varphi(0,t) = \varphi(L,t) = 0, \quad t \in \langle 0,T\rangle, \\
v(0) = v(L) = \psi(0) = \psi(L) = 0, \\
u(x,0) = u_0(x), \quad \frac{\partial u(x,0)}{\partial t} = u_1(x), \quad x \in \langle 0,L\rangle, \\
\varphi(x,0) = \varphi_0(x), \quad \frac{\partial \varphi(x,0)}{\partial t} = \varphi_1(x), \quad x \in \langle 0,L\rangle
\end{aligned}
\tag{2.4}
$$

hold, where u_0, u_1, φ_0, φ_1 are some fixed functions.

The expressions $(F_1+P_1(\varphi),v)$, $(F_2+P_2(\varphi),\psi)$ correspond to the energy of gravitation and wind, respectively. The functions $F_1(x)$, $F_2(x)$ defined on $\langle 0,L\rangle$ represent the gravitational forces and the moment of gravitational forces per a unit length of the center span. If there are no additional weights placed along the center span, then $F_2(x) = 0$ due to the symmetry of the center span.

The functions $P_1(y,x,t)$, $P_2(z,x,t)$ defined on $\mathbb{R} \times \langle 0,L\rangle \times \langle 0,T\rangle$ represent the same quantities of wind forces. These quantities generally depend on φ as is depicted in Fig. 6. In detail the model is studied in [12], where damping forces are considered. These forces are small and we neglect them in our considerations.

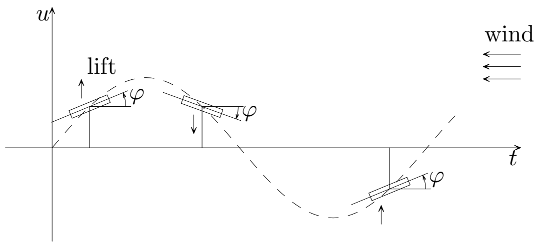

Figure 6.

If the center span is not under the influence of wind and is motionless, it can be described by the steady state equations

$$
\begin{aligned}
k_1(u,v) + b((u+l\varphi)^+,v) + b((u-l\varphi)^+,v) = (F_1,v), \\
k_2(\varphi,\psi) + lb((u+l\varphi)^+,\psi) - lb((u-l\varphi)^+,\psi) = (F_2,v),
\end{aligned}
\tag{2.5}
$$

where u, v, φ, ψ defined on $\langle 0,L\rangle$ are sufficiently smooth. The functions u, v are a solution to (2.5) if the equations are fulfilled for all v, ψ and the boundary conditions

$$
u(0) = u(L) = v(0) = v(L) = \varphi(0) = \varphi(L) = \psi(0) = \psi(L) = 0
\tag{2.6}
$$

hold.

The cable stayed bridge is constructed so that if the center span is exposed to gravitation and is motionless, then the cables are tightened and the solution to (2.5), (2.6) satisfies the relations

$$u(x_i) \mp l\varphi(x_i) \geqslant \varepsilon, \quad i = 1, \dots 8, \tag{2.7}$$

where x_i are the points at which the cables are fixed to the center span and ε is a positive constant. Then u, φ are solutions of the linear system of variational equations

$$\begin{aligned} k_1(u, v) + 2b(u, v) &= (F_1, v), \\ k_2(\varphi, \psi) + 2l^2 b(\varphi, \psi) &= (F_2, v), \end{aligned} \tag{2.8}$$

where u, φ, v, ψ satisfy (2.6).

Wind forces are usually much weaker than gravitational forces. The magnitude of wind is described in [13] by the special norm for $P_1(y, x, t)$, $P_2(y, x, t)$. We can say that if the velocity of the wind is small and the wind slowly changes in time, then the above norm is small too.

If P_1, P_2 are sufficiently small in the norm mentioned above and the initial conditions do not differ very much from the conditions

$$\begin{aligned} u(x, 0) &= u_0(x), \quad \frac{\partial u(x, 0)}{\partial t} = 0, \quad x \in \langle 0, L \rangle, \\ \varphi(x, 0) &= \varphi_0(x), \quad \frac{\partial \varphi(x, 0)}{\partial t} = 0, \quad x \in \langle 0, L \rangle, \end{aligned} \tag{2.9}$$

where u_0, φ_0 are solutions to (2.8), then the cables do not loosen, which is expressed by the inequalities

$$u(x_i, t) \pm l\varphi(x_i, t) \geqslant 0, \quad i = 1, \dots 8, \ \ t \in \langle 0, T \rangle. \tag{2.10}$$

Moreover, the solutions u, φ to (2.3) satisfy the linearized system

$$\begin{aligned} m_1\Big(\frac{\partial^2 u}{\partial t^2}, v\Big) + k_1(u, v) + 2b(u, v) &= (F_1 + P_1(\varphi), v), \\ m_2\Big(\frac{\partial^2 \varphi}{\partial t^2}, \psi\Big) + k_2(\varphi, \psi) + 2l^2 b(\varphi, \psi) &= (F_2 + P_2(\varphi), \psi) \end{aligned} \tag{2.11}$$

with the corresponding boundary and initial conditions.

The above problem was studied in [13] for the model with one row of cables, but everything remains correct for the model with two rows of cables.

Similar problems for some models of suspension bridges were studied in [2], [3], [10].

In the sequel we will cope with a simpler problem described by the equations (2.11) with the right-hand sides

$$F_1(x) = F(x), \quad P_1(\varphi, x, t) = B(x)\varphi, \qquad F_2(x) = 0, \quad P_2(\varphi, x, t) = 0, \tag{2.12}$$

where $F \in L^2((0, L))$ and $B \in L^\infty((0, L))$. Such expressions for forces are realistic for many concrete center spans (see [8]). Moreover, we can say that P_1 is small in the norm mentioned above if B is small in the norm in $L^\infty((0, L))$.

The problem (2.11), (2.12) describes the behaviour of the center span suspended by two rows of the cables which do not loosen, the state of the center span at $t = 0$ does not differ very much from (2.9), and the wind does not change in time.

We will look for the sources of the instabilities connected with the linear behaviour and that is why we will deal with the linear problem (2.11), (2.12) which we analyze in the sections below.

Similar problems can be studied for the model of the center span suspended by one row of cables. For this model the equations (2.3) have to be changed to

$$\begin{aligned} m_1\Big(\frac{\partial^2 u}{\partial t^2}, v\Big) + k_1(u, v) + b(u^+, v) &= (F_1 + P_1(\varphi), v), \\ m_2\Big(\frac{\partial^2 \varphi}{\partial t^2}, \psi\Big) + k_2(\varphi, \psi) &= (F_2 + P_2(\varphi), \psi) \end{aligned} \tag{2.13}$$

and the equations (2.5) to

$$\begin{aligned} k_1(u, v) + b(u^+, v) &= (F_1, v), \\ k_2(\varphi, \psi) &= (F_2, v). \end{aligned} \tag{2.14}$$

The shape of the linear steady state equations connected with (2.8) is clear and the inequalities (2.7), (2.10) have to be changed to

$$\begin{aligned} u(x_i) &\geqslant \varepsilon, \quad i = 1, \dots 8, \\ u(x_i, t) &\geqslant 0, \quad i = 1, \dots 8, \ \ t \in \langle 0, T\rangle. \end{aligned} \tag{2.15}$$

The linear equations (2.11) have to be changed to

$$\begin{aligned} m_1\Big(\frac{\partial^2 u}{\partial t^2}, v\Big) + k_1(u, v) + b(u, v) &= (F_1 + P_1(\varphi), v), \\ m_2\Big(\frac{\partial^2 \varphi}{\partial t^2}, \psi\Big) + k_2(\varphi, \psi) &= (F_2 + P_2(\varphi), \psi) \end{aligned} \tag{2.16}$$

with the right-hand sides (2.12). The corresponding boundary and initial conditions remain the same for the modified equations.

3. Analysis of the linear equations and a criterion of stability

The main goal of this section is to analyze solutions to the equations (2.11), (2.12) which describe the behaviour of the center span in a stable and relatively weak wind.

Let us denote

$$V_1 = H^2((0,L)) \cap H_0^1((0,L)), \quad V_2 = H_0^1((0,L)),$$

where $H_0^1((0,L))$, $H^2((0,L))$ are the Sobolev spaces of functions from $L^2((0,L))$ whose first and second derivatives belong to $L^2((0,L))$. The fact that $u \in H_0^1((0,L))$ means $u(0) = u(L) = 0$. V_1 and V_2 are Hilbert spaces equipped with scalar products

$$\begin{aligned} \langle u, v\rangle_{V_1} &= \int_0^L \{uv + u'v' + u''v''\}\,\mathrm{d}x, \\ \langle \varphi, \psi\rangle_{V_2} &= \int_0^L \{\varphi\psi + \varphi'\psi'\}\,\mathrm{d}x. \end{aligned}$$

By virtue of (2.1) and the Poincaré inequality (see [6]) we have

$$C\|u\|_{V_1}^2 \leqslant k_1(u,u), \quad C\|\varphi\|_{V_2}^2 \leqslant k_2(\varphi,\varphi), \tag{3.1}$$

where C is a positive constant independent of u, φ.

Let us formulate the eigenvalue problems connected with the differential equations (2.11), (2.12). The eigenvalue problem for (2.11) reads

$$\begin{aligned} \alpha m_1(u,v) &= k_1(u,v) + 2b(u,v), \\ \beta m_2(\varphi,\psi) &= k_2(\varphi,\psi) + 2l^2 b(\varphi,\psi). \end{aligned} \tag{3.2}$$

We look for real numbers α, β and functions u, φ from V_1, V_2 such that the equations (3.2) are fulfilled for all v, ψ from V_1, V_2.

Theorem 3.1. *There exist two sequences of eigenvalues $0 < \alpha_1 \leqslant \alpha_2 \ldots$, $0 < \beta_1 \leqslant \beta_2 \ldots$ and eigenfunctions u_n, φ_n of the system (3.2) satisfying*

$$\begin{aligned} m_1(u_m,u_n) &= k_1(u_m,u_n) + 2b(u_m,u_n) = 0, \\ m_2(\varphi_m,\varphi_n) &= k_2(\varphi_m,\varphi_n) + 2l^2 b(\varphi_m,\varphi_n) = 0 \end{aligned}$$

if $m \neq n$, and

$$m_1(u_n,u_n) = m_2(\varphi_m,\varphi_m) = 1.$$

Moreover, u_n is a basis in V_1, $L^2((0,L))$ and φ_n is a basis in V_2, $L^2((0,L))$.

Proof. We will prove the results of the theorem for the first equation in (3.2). The proof for the second equation is similar. Let us define an operator $A\colon L^2(\!(0,L)\!) \to V_1$ such that $A(u) = z$ which is a solution to the variational equation

$$k_1(z,v) + 2b(z,v) = m_1(u,v). \tag{3.3}$$

The equation (3.3) is uniquely solvable, which follows from (3.1) and the continuity of $k_1(\cdot,\cdot)$, $b(\cdot,\cdot)$ on V_1. The continuity of $b(\cdot,\cdot)$ follows from its definition and the fact that $H^1(\!(0,L)\!)$ is continuously embedded into $C^1(\langle 0,L\rangle)$. Hence the operator A is linear and continuous. The restriction of A onto V_1 is a compact operator, which follows from the compactness of the embedding of V_1 into $L^2(\!(0,L)\!)$. If we replace the usual scalar product on V_1 with the equivalent bilinear form $k_1(\cdot,\cdot) + 2b(\cdot,\cdot)$, we have

$$\begin{aligned} k_1(A(u),v) + 2b(A(u),v) &= m_1(u,v), \\ k_1(A(v),u) + 2b(A(v),u) &= m_1(v,u). \end{aligned} \tag{3.4}$$

The equations (3.4) hold for all $u, v \in V_1$ and the bilinear forms $k_1(\cdot,\cdot)$, $b(\cdot,\cdot)$ are symmetric and positive, which yields that the operator A is self-adjoint and positive.

If we apply the theory of linear compact operators (see [19]) and notice that α is an eigenvalue of (3.2) if and only if $1/\alpha$ is an eigenvalue of A, we obtain the desired result. $\square$

We can formulate the eigenvalue problem connected with vertical and torsional oscillations of the center span suspended by one row of cables and prove a theorem similar to Theorem 3.1.

If we look at the equations (2.11) with the right-hand sides (2.12) and take into account Theorem 3.1, we see that the solution of the second equation can be written as the sum

$$\sum_{i=1}^{\infty} a_i \sin\bigl(\sqrt{\beta_i}t + \theta_i\bigr)\varphi_i, \tag{3.5}$$

where β_i, φ_i are the eigenvalues and eigenfunctions of torsional vibrations and the numbers a_i, θ_i can be obtained from the initial conditions by the Fourier method (see [17]).

The expressions in the sum (3.5) generate the periodic forces

$$a_i \sin\bigl(\sqrt{\beta_i}t + \theta_i\bigr)B\varphi_i$$

which act on the center span in the vertical direction due to the structure of the right-hand side (2.12).

If β_i is an eigenvalue of torsional vibrations which is equal or very near to an eigenvalue α_j of vertical vibrations, the resonance can take place. On the other hand, the resonance depends on the number a_i in the sum (3.5). If the number vanishes, the resonance does not take place. We see that the instability by stable wind at the jth vertical mode depends on the initial conditions which are generally the result of a gust of wind.

The same analysis can be made for the model with one row of cables.

A dangerous situation seems to occur if there are eingenvalues β_i, α_j among the first m, n eingevalues of torsional and vertical vibrations, respectively. The values of m, n should be discussed with an expert, but the most dangerous situation seems to correspond to $m = n = 1$.

Keeping the above analysis in mind, we can formulate the following stability criterion.

Criterion S. The larger the numbers

$$\frac{|\alpha_i - \beta_j|}{\alpha_i + \beta_j} \tag{3.6}$$

for $i = 1, \dots m$, $j = 1, \dots n$ are, the more stable the center span is.

4. Some auxiliary results

Before we start dealing with the relationship of eigenvalues and eigenfunctions to some parameters of the center span and the cables, let us prove some facts about compact, self-adjoint, positive linear operators on Hilbert spaces.

Let V be a vector space equipped with a sequence of scalar products $\langle\cdot,\cdot\rangle_i$, $i = 0, 1, \dots$. The space V with these products forms a sequence of Hilbert spaces V_i whose norms we denote by $\|\cdot\|_i$. Let A^i be a sequence of compact, self-adjoint, positive linear operators on V_i which satisfy the following assumptions:

1. $\exists\, C_1, C_2 > 0 \quad \forall\, i \in 1, 2 \dots \ \forall\, u \in V$:
$$C_1\|u\|_0 \leqslant \|u\|_i \leqslant C_2\|u\|_0.$$
2. $\forall\, u \in V$:
$$\lim_{i\to\infty} \|A^i(u) - A^0(u)\|_0 = 0.$$
3. Let u^i in V, $i = 1, 2 \dots$ be a sequence bounded in $\|\cdot\|_0$, then for any subsequence u^l of u^i there exists a subsequence u^j of u^l such that
$$\lim_{j\to\infty} \|A^j(u^j) - A^0(u^j)\|_0 = 0.$$

Before we formulate the main theorem about the operators A^i, let us prove three auxiliary lemmas.

Lemma 4.1. *Let A be a compact, self-adjoint, positive linear operator on the Hilbert space V with the scalar product $\langle\cdot,\cdot\rangle$ and the norm $\|\cdot\|$. Let $u \in V$, $\|u\| = 1$, $\mu \in \mathbb{R}$, and let the inequality*

$$\|A(u) - \mu u\| \leqslant d \tag{4.1}$$

hold, where $0 < d < 1$. Then there exists an eigenvalue λ_i such that

$$|\lambda_i - \mu| \leqslant \sqrt{d}. \tag{4.2}$$

If $J \subset N$ is the set of all i's such that λ_i satisfy the inequality (4.2), *then there exist $\alpha_i \in \mathbb{R}$ such that the estimate*

$$\Bigl\|u - \sum_{i\in J} \alpha_i u_i\Bigr\| \leqslant \sqrt{d} \tag{4.3}$$

holds, where u_i are the eigenvectors corresponding to λ_i.

Proof. If A is a compact, self-adjoint, positive linear operator, there exist eigenvalues $\lambda_1 \geqslant \lambda_2 \ldots > 0$ and eigenvectors u_i which form an orthonormal basis. Then we have

$$d^2 \geqslant \|A(u) - \mu u\|^2 = \Bigl\|\sum_{i=1}^{\infty} \lambda_i \langle u_i, u\rangle u_i - \mu \langle u_i, u\rangle u_i\Bigr\|^2 = \sum_{i=1}^{\infty} (\lambda_i - \mu)^2 \langle u_i, u\rangle^2. \tag{4.4}$$

If the inequality $|\lambda_i - \mu| > \sqrt{d}$ holds for all i, then (4.4) implies

$$d^2 \geqslant d \sum_{i=1}^{\infty} \langle u_i, u\rangle^2 = d. \tag{4.5}$$

The relation (4.5) contradicts the facts that $\|u\| = 1$ and $0 < d < 1$. From (4.4) and the definition of J we obtain

$$d^2 \geqslant d \sum_{i\in N-J}^{\infty} \langle u_i, u\rangle^2 = d \Bigl\|u - \sum_{i\in J} \langle u_i, u\rangle u_i\Bigr\|^2. \tag{4.6}$$

If we substitute $\langle u_i, u\rangle$ for α_i in (4.3) we have the desired result. $\square$

Let λ_k^i, N_k^i denote the kth eigenvalue of A^i and the subspace of all eigenfunctions corresponding to this eigenvalue. Moreover, let the inequalities

$$\lambda_1^i \geqslant \lambda_2^i \geqslant \ldots > 0$$

hold, where multiplicity is taken into account. Thus if for s, i, k the relations $\lambda_{k-1}^i > \lambda_k^i = \ldots = \lambda_{k+s}^i > \lambda_{k+s+1}^i$ are fulfilled, then $N_k^i = \ldots = N_{k+s}^i$.

Lemma 4.2. *Let a sequence A^i, $i = 0, 1 \ldots$ of operators fulfill Assumptions* 1–3. *If α is an eigenvalue of A^0, then there exists a sequence $\lambda^i_{k_i}$ such that*

$$\lim_{i\to\infty} \lambda^i_{k_i} = \alpha. \tag{4.7}$$

Proof. Let α be an eigenvalue and u an eigenvector of A^0, then Assumption 2 yields

$$\lim_{i\to\infty} \|A^i(u) - \alpha u\|_0 = 0. \tag{4.8}$$

From Lemma 4.1, Assumption 1, and (4.8) the existence of a sequence $\lambda^i_{k_i}$ which satisfies (4.7) follows. □

Lemma 4.3. *Let a sequence A^i, $i = 0, 1 \ldots$ of operators fulfill Assumptions* 1–3. *If $\lambda^i_{k_i}$ is a sequence of eigenvalues of A^i and α is a limit point of this sequence, then α is an eigenvalue of A^0.*

Proof. Let α be a limit point of $\lambda^i_{k_i}$, then there exists a subsequence $\lambda^j_{k_j}$ such that

$$\lim_{j\to\infty} \lambda^j_{k_j} = \alpha. \tag{4.9}$$

Assumption 3 yields the existence of a subsequence $\lambda^l_{k_l}$ and the corresponding eigenvectors u^l such that

$$\lim_{l\to\infty} \|\lambda^l_{k_l} u^l - A(u^l)\|_0 = 0. \tag{4.10}$$

From (4.9), (4.10), and Lemma 4.1 the assertion of the lemma follows. □

Theorem 4.1. *If A^i is a sequence of operators on V satisfying Assumptions* 1–3, *then for any k the limit relation*

$$\lim_{i\to\infty} \lambda^i_k = \lambda^0_k \tag{4.11}$$

holds.

Proof. First, we prove (4.11) for $k = 1$. From Lemma 4.2 it follows that there exists a sequence $\lambda^i_{k_i}$ which converges to λ^0_1. If we notice that $\lambda^i_1 \geqslant \lambda^i_{k_i}$, then Lemma 4.3 implies that the sequence λ^i_1 has a single limit point λ^0_1 and converges to λ^0_1.

Second, let (4.11) hold for all $k \leqslant n$, then we prove (4.11) for $k = n+1$. We will study two different cases $\lambda^0_{n+1} < \lambda^0_n$ and $\lambda^0_{n+1} = \lambda^0_n$.

Let us start with the case $\lambda^0_{n+1} < \lambda^0_n$. Lemma 4.2 yields that there exists a sequence $\lambda^i_{k_i}$ which converges to λ^0_{n+1}.

The inequality $\lambda^0_{n+1} < \lambda^0_n$ yields that $n+1 \leqslant k_i$ for all sufficiently large i. From the last inequality and Lemma 4.3 it follows that the sequence λ^i_{n+1} has at most two limit points λ^0_n, λ^0_{n+1}.

We prove indirectly that λ^i_{n+1} converges to λ^0_{n+1}. Let us assume that there exists a subsequence λ^j_{n+1} of λ^i_{n+1} which converges to λ^0_n and the relations

$$\lambda^0_m > \lambda^0_{m+1} = \ldots = \lambda^0_n \tag{4.12}$$

hold for some m such that $m < n$. Then the convergence of λ^j_{n+1} to λ^0_n and the relations (4.12) yield that there exist sequences u^j_k for $k = m+1, \ldots, n+1$ such that

$$u^j_k \in N^j_k, \quad \|u^j_k\|_j = 1, \quad \langle u^j_k, u^j_l \rangle_j = 0 \tag{4.13}$$

if $k \neq l$, $k, l = m+1, \ldots, n+1$.

Moreover, if we take into account Assumption 3, we can choose sequences such that the limit relations

$$\lim_{j\to\infty} \|A^j(u^j_k) - A^0(u^j_k)\|_0 = 0, \quad k = m+1, \ldots, n+1 \tag{4.14}$$

hold.

From the limits (4.14) and Lemma 4.1 it follows that there exist sequences v^j_k, $k = m+1, \ldots, n+1$ from N^0_n such that the limit relations

$$\lim_{j\to\infty} \|u^j_k - v^j_k\|_0 = 0, \quad k = m+1, \ldots, n+1 \tag{4.15}$$

hold.

If we take into account Assumption 1, (4.13) and (4.15), we see that the dimension of N^0_n is $n+1-m$, which is a contradiction.

Let us continue with the case $\lambda^0_{n+1} = \lambda^0_n$. We will prove (4.11) indirectly again. Let us assume that the sequence λ^i_{n+1} does not converge to λ^0_{n+1}, then there exists a subsequence λ^j_{n+1} of λ^i_{n+1} such that

$$\lim_{j\to\infty} \lambda^j_{n+1} = \alpha < \lambda^0_{n+1}. \tag{4.16}$$

Let us assume that the relations

$$\lambda^0_m > \lambda^0_{m+1} = \ldots \lambda^0_n = \lambda^0_{n+1} \tag{4.17}$$

hold for some m, where $m < n$.

If we take any $u \in N^0_{n+1}$ and note that $\lim\limits_{i\to\infty} \|A^j(u) - A^0(u)\|_0 = 0$, which follows from Assumption 2, then Lemma 4.1 and (4.16) yield the existence of a sequence v^j which satisfies

$$v^j \in N^j_{m+1} + \ldots + N^j_n, \quad \lim_{j\to\infty} \|u - v^j\|_j = 0. \tag{4.18}$$

From (4.17), (4.18), and Assumption 1 it follows that the dimension of N^0_{n+1} is equal to the dimension $N^j_{m+1} + \ldots + N^j_n$ for sufficiently large j, which is a contradiction. □

5. Continuous dependence of eigenvalues on some parameters and one optimal problem

We will exploit the results from Section 4 to express the relationship between the stability of the center span in wind and some parameters connected with the center span and the cables. Let us consider that we can change the positions and stiffness of the cables and place some additional weights along the center span as is depicted in Fig. 7.

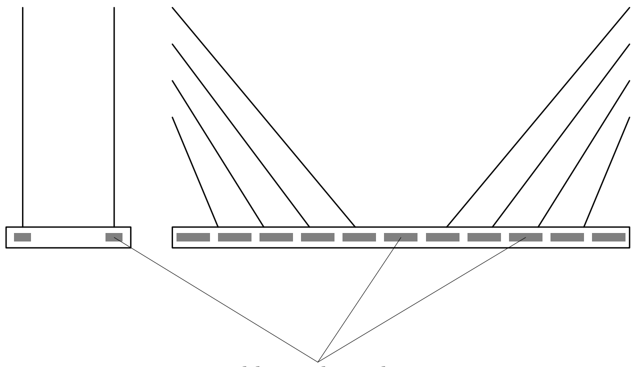

Figure 7.

We see that additional weights change the functions M_1, M_2, while the functions K_1, K_2 remain unchanged. Let us consider sequences M^i_1, M^i_2, k^i_j, x^i_j, $j = 1, \ldots 8$, $i = 1, 2 \ldots$ such that $k^i_j \geqslant 0$, $x^i_j \in \langle 0, L\rangle$ and denote the corresponding bilinear forms by $m^i_1(\cdot,\cdot)$, $m^i_2(\cdot,\cdot)$, $b^i(\cdot,\cdot)$.

Let α^i_l, β^i_l denote the eigenvalues of the variational equations

$$\begin{aligned} \alpha m^i_1(u, v) &= k_1(u, v) + 2b^i(u, v), \\ \beta m^i_2(\varphi, \psi) &= k_2(\varphi, \psi) + 2l^2 b^i(\varphi, \psi). \end{aligned} \tag{5.1}$$

The eigenvalues are ordered increasingly with respect to l. Then the following theorem holds.

Theorem 5.1. *Let C_1, C_2 be positive constants satisfying*

$$C_1 \leqslant M_1^i, \quad M_2^i \leqslant C_2 \tag{5.2}$$

and moreover, let

$$\begin{gathered} M_1^i \rightharpoonup M_1^0, \quad M_2^i \rightharpoonup M_2^0 \quad \text{in } L^2((0,L)), \\ k_j^i \to k_j^0, \quad x_j^i \to x_j^0, \quad j = 1, \dots 8 \quad \text{in } \mathbb{R}, \end{gathered}$$

where the symbol $\rightharpoonup$ denotes the weak convergence. Then

$$\alpha_l^i \to \alpha_l^0, \quad \beta_l^i \to \beta_l^0, \quad l = 1, 2 \dots \tag{5.3}$$

if $i \to \infty$.

P r o o f. We will prove the theorem for the eigenvalues α_l^i of the first equation in (5.1).

Define operators $L^i\colon L^2((0,L)) \to V_1$ by $L^i(u) = v$, where v is the unique solution to the variational equation

$$k_1(v,w) + 2b^i(v,w) = m_1^i(u,w) \tag{5.4}$$

which is satisfied for all $w \in V_1$. The existence, uniqueness, and continuous dependence of v on u follow from the Lax-Milgram theorem. Let us define $A^i = P^i L^i$, where P^i is the embedding from V_1 into $L^2((0,L))$ with the scalar product given by $m_1^i(\cdot,\cdot)$. Since the embedding P^i is compact (see [6]), the operators A^i are compact as well.

For any $u, v \in L^2((0,L))$ we have the equations

$$\begin{aligned} k_1(L^i(u), L^i(v)) + 2b^i(L^i(u), L^i(v)) &= m_1^i(u, A^i(v)), \\ k_1(L^i(v), L^i(u)) + 2b^i(L^i(v), L^i(u)) &= m_1^i(v, A^i(u)). \end{aligned} \tag{5.5}$$

Since the bilinear forms $k_1(\cdot,\cdot) + 2b^i(\cdot,\cdot)$ are symmetric on V_1, the formulas (5.4) yield that A^i are self-adjoint, non-negative operators on $L^2((0,L))$ equipped with the scalar products $m_1^i(\cdot,\cdot)$. From the definition of A^i it follows that α_l^i are eigenvalues of the first equation in (5.1) if and only if $1/\alpha_l^i$ are eigenvalues of A^i.

Theorem 4.1 yields that the assertions of this theorem would be proved if the sequence A^i satisfies Assumptions 1–3.

Assumption 1 follows from the inequalities (5.2).

Let us prove Assumption 2. If v_i is a sequence of solutions to the variational equations

$$k_1(v^i, w) + 2b^i(v^i, w) = m_1^i(u, w) \tag{5.6}$$

which hold for all $w \in V_1$, then the inequalities (3.1) and the equations (5.6) yield the existence of a constant C independent of i such that the estimate

$$C\|v^i\|_{V_1}^2 \leqslant k_1(v^i, v^i) + 2b^i(v^i, v^i) = m_1^i(u, v^i) \tag{5.7}$$

holds. From (5.7) we obtain the estimate $\|v^i\|_{V_1} < C$, where C is a constant independent of i.

Thus there exists a subsequence v^j of v^i which weakly converges to v^* in V_1. If we take into account that V_1 is compactly embedded in $C(\langle 0, L\rangle)$ (see [15]), then the definitions of $b^i(\cdot,\cdot)$ yield

$$b^i(v^i, w) \to b^0(v^*, w), \tag{5.8}$$

which holds for any $w \in V_1$.

From $M_1^i u \rightharpoonup M_1^0 u$ in $L^2((0, L))$, (5.8) and $v^i \rightharpoonup v^*$ in V_1 it follows that v^* is a solution to the variational equation

$$k_1(v^*, w) + 2b^0(v^*, w) = m_1^0(u, w). \tag{5.9}$$

This implies that v^* is equal to the unique solution v^0 to the equation (5.9). Moreover, the whole sequence v^i weakly converges to v^0 in V_1.

Thus $L^i(u) \rightharpoonup L^0(u)$ in V_1 for all u in $L^2((0, L))$ and the definitions of A^i, A^0 yield that $A^i(u) \to A^0(u)$ in $L^2((0, L))$ with the scalar product $m^0(\cdot,\cdot)$, which proves Assumption 2.

Let us prove Assumption 3. If u^i is a bounded sequence in $L^2((0, L))$, then there exists a subsequence u^j which weakly converges to u^0 in $L^2((0, L))$. Consequently, $M_1^j u^j \rightharpoonup M_1^0 u^0$ in $L^2((0, L))$.

Let us consider the sequences v^j, z^j from V_1 which consist of the solutions to the variational equations

$$\begin{aligned} k_1(v^j, w) + 2b^j(v^j, w) &= m_1^j(u^j, w), \\ k_1(z^j, w) + 2b^0(z^j, w) &= m_1^0(u^j, w). \end{aligned} \tag{5.10}$$

Then if we follow the ideas of the proof of Assumption 2, we can prove that the sequences v^j, z^j of solutions to (5.10) weakly converge to v^0 in V_1 which is the solution to the variational equation (5.9), where u is substituted for u^0.

The last fact yields $L^j(u^j) \rightharpoonup L^0(u^0)$, $L^0(u^j) \rightharpoonup L^0(u^0)$ in V_1. The definitions of A^j, A^0 yield $A^j(u^j) \to A^0(u^0)$, $A^0(u^j) \to A^0(u^0)$ in $L^2((0, L))$, which proves Assumption 3. $\square$

The above theorem shows that the eigenvalues continuously depend on the positions and stiffness of cables and on the additional weights.

We will solve an optimal problem in which we look for the parameters of cable systems and the values of additional weights which could guarantee a certain stability in wind.

Let

$$\mathcal{K}^1 \subset \langle 0, L\rangle \times \ldots \times \langle 0, L\rangle \subset \mathbb{R}^8, \quad \mathcal{K}^2 \subset \langle 0, K\rangle \times \ldots \times \langle 0, K\rangle \subset \mathbb{R}^8$$

be sets which are compact and whose elements are denoted by $x = (x_1, \ldots, x_8)$, $k = (k_1, \ldots, k_8)$. Moreover, let

$$\mathcal{U}^1 \subset L^2((0, L)), \quad \mathcal{U}^2 \subset L^2((0, L))$$

be convex, closed sets of functions from $L^2((0, L))$ which satisfy the inequalities (5.2). Functions M_1, M_2 from these sets correspond to the permissible values of additional weights. For m, n from N we can define the functional

$$J_{m,n}(M_1, M_2, x, k) = \min_{\substack{1 \leqslant i \leqslant m, \\ 1 \leqslant j \leqslant n}} \frac{|\alpha_i - \beta_j|}{\alpha_i + \beta_j}.$$

Now we can solve the optimal problem

$$\max_{\substack{M_1 \in \mathcal{U}^1,\, M_2 \in \mathcal{U}^2, \\ x \in \mathcal{K}^1,\, k \in \mathcal{K}^2}} J_{m,n}(M_1, M_2, x, k). \tag{5.11}$$

The number (5.11) corresponds to Criterion S and minimizes some instabilities of the behaviour of the center span in wind.

Theorem 5.2. *There exist $M_1 \in \mathcal{U}^1$, $M_2 \in \mathcal{U}^2$, $x \in \mathcal{K}^1$, $k \in \mathcal{K}^2$ which maximize the functional $J_{m,n}$.*

P r o o f. Let M_1^i, M_2^i, x^i, k^i be maximization sequences for $J_{m,n}$. By virtue of the definitions of $\mathcal{U}^1$, $\mathcal{U}^2$, $\mathcal{K}^1$, $\mathcal{K}^2$, we can select subsequences M_1^j, M_2^j, x^j, k^j such that $M_1^j \rightharpoonup M_1^0$, $M_2^j \rightharpoonup M_2^0$ in $L^2((0, L))$ and $x^j \to x^0$, $k^j \to k^0$ in $\mathbb{R}^8$ and moreover, $M_1^0 \in \mathcal{U}^1$, $M_2^0 \in \mathcal{U}^2$, $x^0 \in \mathcal{K}^2$, $k^0 \in \mathcal{K}^2$. Theorem 5.1 yields that $J_{m,n}$ achieves its maximum at M_1^0, M_2^0, x^0, k^0. $\square$

So far we have studied the central span suspended by one or two rows of cables and each row contained eight cables. In reality the center span is suspended by much greater number of cables placed densely along the center span.

There is a question whether it is possible to change the cables to a continuous medium. This process can be described by the b-h convergence proposed in [12], [13]. Let us recall the definition, but first of all let us define a bilinear form

$$h(u,v)=\int_0^L Guv\,\mathrm{d}x,$$

where $G\in L^\infty((0,L))$.

Definition 6.1. Let n_i be an increasing sequence of natural numbers, let $\{x_j^i\}_{j=1}^{n_i}$, $\{k_j^i\}_{j=1}^{n_i}$ satisfy $0\leqslant x_1^i\leqslant x_2^i\leqslant\ldots x_{n_i}^i\leqslant L$, $k_j^i\geqslant 0$ for all $i=1,2\ldots$, $j=1,\ldots,n_i$ and let G from $L^\infty((0,L))$ satisfy $G(x)\geqslant 0$ for all x in $(0,L)$. Then $\{x_j^i\}_{j=1}^{n_i}$, $\{k_j^i\}_{j=1}^{n_i}$ b-h converge to G if the relation

$$\lim_{i\to\infty}\sum_{j=1}^{n_i}k_j^i f(x_j^i)=\int_0^L Gf\,\mathrm{d}x$$

holds for all $f\in C(\langle 0,L\rangle)$. Moreover, let us denote

$$b^i(u,v)=\sum_{j=1}^{n_i}k_j^i u(x_j^i)v(x_j^i).$$

This definition describes the process of gradual substitution of one cable system by another one with a greater number of thinner cables. Let us consider the sequence of variational equations

$$\begin{aligned}k_1(u^i,v)+2b^i(u^i,v)&=(F_1,v),\\ k_2(\varphi^i,\psi)+2l^2b^i(\varphi^i,\psi)&=(F_2,\psi),\end{aligned}\tag{6.1}$$

where $F_1,F_2\in L^2((0,L))$. Then u^i, φ^i from V_1, V_2 are solutions to (6.1) if these equations are fulfilled for all v, ψ from V_1, V_2. Further, let us consider the variational equations

$$\begin{aligned}k_1(u,v)+2h(u,v)&=(F_1,v),\\ k_2(\varphi,\psi)+2l^2h(\varphi,\psi)&=(F_2,\psi).\end{aligned}\tag{6.2}$$

The relation between the solutions of (6.1) and the solution of (6.2) is given by the following theorem whose proof can be found in [12].

Theorem 6.1. *Let* $\{x_j^i\}_{j=1}^{n_i}$, $\{k_j^i\}_{j=1}^{n_i}$ *b-h converge to* G. *If* u^i, φ^i *are solutions to* (6.1) *and* u^0, φ^0 *are a solution to* (6.2), *then*

$$\begin{aligned} u^i &\to u^0 \quad \textit{in } V_1, \\ \varphi^i &\to \varphi^0 \quad \textit{in } V_2. \end{aligned}$$

We can study the variational equations

$$\begin{aligned} k_1(u,v) + 2b^i(u,v) &= \alpha m_1(u,v), \\ k_2(\varphi,\psi) + 2l^2 b^i(\varphi,\psi) &= \beta m_2(\varphi,\psi) \end{aligned} \tag{6.3}$$

for the eigenvalue problems with the individual cables and the variational equations

$$\begin{aligned} k_1(u,v) + 2h(u,v) &= \alpha m_1(u,v), \\ k_2(\varphi,\psi) + 2l^2 h(\varphi,\psi) &= \beta m_2(\varphi,\psi) \end{aligned} \tag{6.4}$$

for the eigenvalue problem with the homogenized cable system.

Let α_k^i, β_k^i denote the eigenvalues of (6.3) and α_k^0, β_k^0 the eigenvalues of (6.4). Then we have the following theorem.

Theorem 6.2. *Let* $\{x_j^i\}_{j=1}^{n_i}$, $\{k_j^i\}_{j=1}^{n_i}$ *b-h converge to* G. *Then*

$$\alpha_k^i \to \alpha_k^0, \quad \beta_k^i \to \beta_k^0$$

for all k.

P r o o f. We restrict ourselves to eigenvalues α_k^i. Let us assume that $L^2((0,L))$ is equipped with the norm given by the scalar product $m_1(\cdot,\cdot)$. The proof of this theorem is parallel to the proof of Theorem 5.1. The sequence of Hilbert spaces in the proof of Theorem 5.1 is replaced by the constant sequence of $L^2((0,L))$ with the norm derived from the scalar product $m_1(\cdot,\cdot)$. The sequence of the operators A^i is defined by the relation $A^i = PL^i$, where P is the imbedding of V_1 into $L^2((0,L))$ and $L^i\colon L^2((0,L)) \to V_1$ assigns $u \in L^2((0,L))$ to the unique solution v of the variational equation

$$k_1(v,w) + 2b^i(v,w) = m_1(u,w).$$

The operator A^0 is defined by the relation $A^0 = PL^0$, where P is the same imbedding from V_1 to $L^2((0,L))$ as above and $L^0\colon L^2((0,L)) \to V_1$ assigns $u \in L^2((0,L))$ to the unique solution v of the variational equation

$$k_1(v,w) + 2h(v,w) = m_1(u,w).$$

If we apply Theorem 4.1, we can follow the proof of Theorem 5.1. $\square$

7. Two special examples

In this section we will deal with two special model problems connected with the two basic models of the center span suspended by one or two rows of cables. Let us assume that the cables are distributed with sufficient density so that the homogenized model from Section 6 can be applied. Moreover, the functions $M_1(x)$, $M_2(x)$, $K_1(x)$, $K_2(x)$, $G(x)$ which characterize the center span and the cables are constant and we denote them by M_1, M_2, K_1, K_2, G.

Let us start with the model of the center span suspended by two rows of cables, then the system (6.4) can be rewritten to the system of ordinary differential equations

$$\begin{aligned} \frac{K_1 u^{(4)}}{M_1} + \frac{2Gu}{M_1} &= \alpha u, \\ -\frac{K_2 \varphi''}{M_2} + \frac{2l^2 G\varphi}{M_2} &= \beta\varphi \end{aligned} \tag{7.1}$$

with the boundary conditions

$$\begin{aligned} u(0) = u(L) = u''(0) = u''(L) = 0, \\ \varphi(0) = \varphi(L) = 0. \end{aligned} \tag{7.2}$$

The eigenvalues and eigenfunctions can be explicitly calculated and we have

$$\begin{aligned} \alpha_n &= \frac{K_1 \pi^4 n^4}{M_1 L^4} + \frac{2G}{M_1}, \qquad u_n(x) = \sin\Big(\frac{\pi n x}{L}\Big), \\ \beta_n &= \frac{K_2 \pi^2 n^2}{M_2 L^2} + \frac{2l^2 G}{M_2}, \quad \varphi_n(x) = \sin\Big(\frac{\pi n x}{L}\Big). \end{aligned} \tag{7.3}$$

The model of the center span suspended with one row of cables is described by the system of ordinary differential equations

$$\begin{aligned} \frac{K_1 u^{(4)}}{M_1} + \frac{Gu}{M_1} &= \alpha u, \\ -\frac{K_2 \varphi''}{M_2} &= \beta\varphi \end{aligned} \tag{7.4}$$

with the boundary conditions (7.2)

The eigenvalues and eigenfunctions can be explicitly calculated again and we have

$$\begin{aligned} \alpha_n &= \frac{K_1 \pi^4 n^4}{M_1 L^4} + \frac{G}{M_1}, \quad u_n(x) = \sin\Big(\frac{\pi n x}{L}\Big), \\ \beta_n &= \frac{K_2 \pi^2 n^2}{M_2 L^2}, \qquad\qquad \varphi_n(x) = \sin\Big(\frac{\pi n x}{L}\Big). \end{aligned} \tag{7.5}$$

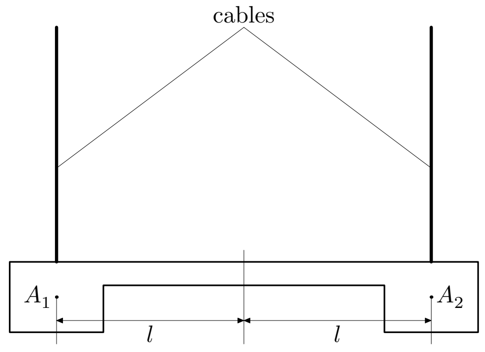

Figure 8.

Let us have a look at Fig. 8 which describes a usual shape of a center span suspended by two rows of cables (see [18]).

If we assume that the mass of the center span is distributed so that the values of M_1, M_2 can be approximated as if the mass were concentrated along the lines run through the points A_1, A_2 and the density of that concentrated mass is $\overline{M}$, then we have

$$M_1(x) = 2\overline{M}, \quad M_2(x) = 2l^2\overline{M}, \tag{7.6}$$

which follows from (2.2).

If we consider (7.3), (7.6) and Criterion S for $m = n = 1$, we have

$$\frac{|\alpha_1 - \beta_1|}{\alpha_1 + \beta_1} = \frac{\left|\dfrac{K_1\pi^4}{2\overline{M}L^2} - \dfrac{K_2\pi^2}{2l^2\overline{M}}\right|}{L^2\left(\dfrac{2G}{\overline{M}} + \dfrac{K_1\pi^4}{2\overline{M}L^4} + \dfrac{K_2\pi^2}{2l^2\overline{M}L^2}\right)}. \tag{7.7}$$

From 7.7 we obtain the limit

$$\lim_{L\to\infty} \frac{|\alpha_1 - \beta_1|}{\alpha_1 + \beta_1} = 0. \tag{7.8}$$

The limit (7.8) indicates that the behaviour of sufficiently long center spans could be instable, which is possibly due to the special distribution of mass along the center spans. The technology of cable stayed bridges makes it possible to erect bridges with long center spans whose properties given by M_1, M_2, K_1, K_2, G are relatively independent of their length; then the limit (7.8) indicates a certain instability of such center spans.

If we explicitly calculate the expression (7.7) applying (7.5) and study the limit (7.8) for the new values, we see that this limit cannot vanish for any special distribution of mass.

This fact seems to indicate that the center span suspended by one row of cables could be more stable in wind.

8. Conclusion

In some technical papers the instability of cable-stayed and suspension bridges is being explained by their aerodynamic properties. The wind passing by the span induces a periodic force, which is connected with the shape of the cross section and its Strouhal's number (see [14]). If the speed of the wind achieves a certain value, the frequency of the wind-induced force corresponds to an eigenvalue of the bridge, which can result in violent oscillations and then in loosening of the cables.

Such an explanation was criticized in [8], where the authors opposed arguing that the frequency of oscillations of the Tacoma Narrows suspension bridge was independent of the speed of wind.

In this paper we have tried to present an explanation based on a certain relation between the eigenvalues of the vertical and torsional oscillations of the center span. Consequently, our explanation is based on some properties connected with the construction of bridges and is independent of the speed of wind.

References

[1] *N. U. Ahmed, H. Harbi*: Mathematical analysis of dynamic models of suspension bridges. SIAM J. Appl. Math. *58* (1998), 853–874.

[2] *J. Berkovits, P. Drábek, H. Leinfelder, V. Mustonen, and G. Tajčová*: Time-periodic oscillations in suspension bridges: Existence of unique solution. Nonlinear Anal., Real World Appl. *1* (2000), 345–362.

[3] *P. Drábek, H. Leinfelder, G. Tajčová*: Coupled string-beam equations as a model of suspension bridges. Appl. Math. *44* (1999), 97–142.

[4] *A. Fonda, Z. Schneider, and F. Zanolin*: Periodic oscillations for a nonlinear suspension bridge model. J. Comput. Appl. Math. *52* (1994), 113–140.

[5] *J. Glover, A. C. Lazer, and P. J. McKenna*: Existence and stability of large-scale nonlinear oscillations in suspension bridges. Z. Angew. Math. Phys. *40* (1989), 171–200.

[6] *A. Kufner, O. John, S. Fučík*: Functional spaces. Academia, Prague, 1977.

[7] *A. C. Lazer, P. J. McKenna*: Large scale oscillatory behaviour in loaded asymmetric systems. Ann. Inst. Henri Poincaré, Anal. Non Linéaire *4* (1987), 243–274.

[8] *A. C. Lazer, P. J. McKenna*: Large-amplitude periodic oscillations in suspension bridges: some new connections with nonlinear analysis. SIAM Rev. *32* (1990), 537–578.

[9] *A. C. Lazer, P. J. McKenna*: Existence, uniqueness, and stability of oscillations in differential equations with asymmetric nonlinearities. Trans. Am. Math. Soc. *315* (1989), 721–739.

[10] *P. J. McKenna, W. Walter*: Nonlinear oscillations in a suspension bridge. Arch. Rational Mech. Anal. *98* (1987), 167–190.

[11] *P. J. McKenna, W. Walter*: Travelling waves in a suspension bridge. SIAM J. Appl. Math. *50* (1990), 703–715.

[12] *J. Malík*: Oscillations in cable stayed bridges: existence, uniqueness, homogenization of cable systems. J. Math. Anal. Appl. *226* (2002), 100–126.

[13] *J. Malík*: Mathematical modelling of cable stayed bridges: existence, uniqueness, homogenization of cable systems. Appl. Math. *49* (2004), 1–38.
[14] *E. Simiu, R. H. Scanlan*: Wind Effects on Structures: An Introduction to Wind Engineering. John Wiley, New York, 1978.
[15] *S. L. Sobolev*: Applications of Functional Analysis in Mathematical Physics. American Mathematical Society, Providence, 1963.
[16] *G. Tajčová*: Mathematical models of suspension bridges. Appl. Math. *42* (1997), 451–480.
[17] *V. S. Vladimirov*: Equations of Mathematical Physics. Mir, Moscow, 1987. (In Italian.)
[18] *R. Walther, B. Houriet, W. Isler, P. Moïa, and J. F. Klein*: Cable Stayed Bridges. Thomas Telford, London, 1999.
[19] *K. Yosida*: Functional Analysis. Springer-Verlag, Berlin-Götingen-Heidelberg, 1965.

Author's address: *J. Malík*, Institute of Geonics, Academy of Sciences of the Czech Republic, Studentská 1768, 708 00 Ostrava, Czech Republic, e-mail: malik@ugn.cas.cz.